

# On the rationality of the OPERA experiment as a signal of Lorentz violation

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We show that the superluminal muon neutrinos in the recent OPERA experiment can exist theoretically. The refutation of the OPERA experiment from some theoretical arguments is not universally valid, but resulting from some implicit assumptions. Our argument can accommodate both the OPERA experiment for superluminal neutrinos and the ICARUS experiment of no evidence for the analogues Cherenkov radiation of muon neutrinos from CERN to the LNS.

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Recently, the OPERA collaboration reported that the speed of muon neutrinos is larger than the vacuum light speed by a factor of  $10^{-5}$  [1]. Such observation has been suggested [2, 3] as a signal for Lorentz violation within several existing frameworks [4–6] of Lorentz violation, and the corresponding Lorentz violation parameters of muon neutrinos are estimated to be of the order of  $10^{-5}$  [2, 3]. Cohen and Glashow argued that if the Lorentz violation of the OPERA experiment is of  $10^{-5}$ , the high energy muon neutrinos exceeding tens of GeVs can not be detected by the Gran Sasso detector, mainly because of the energy-losing process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  analogous to Cherenkov radiations through the long baseline about 730 km [7]. Bi *et al.* also argued that the Lorentz violation of muon neutrinos of order  $10^{-5}$  will forbid kinematically the production process of muon neutrinos  $\pi \rightarrow \mu + \nu_\mu$  for muon neutrinos with energy larger than about 5 GeV [8]. Such arguments put up a strong challenge to the rationality of the OPERA experiment and the consequent suggestion to attribute the OPERA experiment as a signal of Lorentz violation. In the following, with a Lorentz violation framework from basic considerations [4, 5], we discuss the three Cohen-Glashow processes first, and consider also the production processes of Bi *et al.*.

A response to Cohen, Glashow and Bi *et al.* is offered in Ref. [9], where Amelino-Camelia, Freidel, Kowalski-Glikman and Smolin argued that the energy threshold for the anomalously Cherenkov analogous process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  makes physics observer-dependent. They pointed out that the deformed Lorentz transformation can avoid the problems brought about by these arguments. In this paper we propose an alternative argument for the rationality of the OPERA result as a signal for Lorentz violation, and we do not use deformed Lorentz transformations. From the point of view of the Lorentz violation framework, the arguments of Ref. [7]

are just one of theoretical possibilities from some implicit assumptions, which are not easily seen obviously, such as that neutrinos have the fixed Lorentz violation when the three Cherenkov-like processes happen (this is the most implicit there), that all the three generations of neutrinos have the same Lorentz violation and that the Lorentz violation is described enough by a scalar parameter. When we work in any Lorentz invariance violation frameworks, such as the standard model supplement (SMS) [4, 5] and the standard model extension (SME) [6], the previously well known physical regulations gotten in the case of Lorentz invariance become subtle. Since the Lagrangian in the Lorentz violation framework does not contain explicitly space-time coordinates, so the translation invariance of space-time of the Lagrangian is kept. Then the 4-momenta conservation law is also applicable. The question whether the process is kinematically forbidden can be discussed. We assume that the 4-momenta  $p$  of real particles are time-like, i.e.  $p^2 \geq 0$  and the time-component of  $p$  is non-negative.

The kinematic limit for the process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  is

$$|p_{\nu_\mu, i}| \geq |p_{\nu_\mu, f}| + |p_{e^+}| + |p_{e^-}|. \quad (1)$$

Without Lorentz violation, the mass-energy relation is  $p^2 = m^2$ . Then Eq. (1) becomes

$$m_{\nu_\mu} \geq m_{\nu_\mu} + 2m_e. \quad (2)$$

It is obvious that Eq. (2) can not be satisfied. So the Cherenkov analogous process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  is forbidden kinematically in space-time in the case of no Lorentz violation. We use here a Lorentz violation framework, e.g. the SMS [4, 5], in which there is a replacement of the ordinary partial  $\partial_\alpha$  and the covariant derivative  $D_\alpha$  by  $\partial^\alpha \rightarrow M^{\alpha\beta} \partial_\beta$  and  $D^\alpha \rightarrow M^{\alpha\beta} D_\beta$ , where  $M^{\alpha\beta}$  is a local matrix. We separate it to two matrices like  $M^{\alpha\beta} = g^{\alpha\beta} + \Delta^{\alpha\beta}$ , where  $g^{\alpha\beta}$  is the metric tensor of

space-time. Since  $g^{\alpha\beta}$  is Lorentz invariant,  $\Delta^{\alpha\beta}$  contains all the Lorentz violating degrees of freedom from  $M^{\alpha\beta}$ . Then  $\Delta^{\alpha\beta}$  brings new terms violating Lorentz invariance in the standard model and is called Lorentz invariance violation matrix therefore. The mass-energy relation for electrons and neutrinos becomes [2, 4]

$$p^2 + g_{\alpha\mu}\Delta^{\alpha\beta}\Delta^{\mu\nu}p_\beta p_\nu + 2\Delta^{\alpha\beta}p_\alpha p_\beta - m^2 = 0. \quad (3)$$

So

$$p^2 = m^2 + \lambda(\Delta, p), \quad (4)$$

where

$$\begin{aligned} \lambda(\Delta, p) &\equiv -g_{\alpha\mu}\Delta^{\alpha\beta}\Delta^{\mu\nu}p_\beta p_\nu - 2\Delta^{\alpha\beta}p_\alpha p_\beta \\ &= -p^t G \Delta^t G \Delta G p - 2p^t G \Delta G p. \end{aligned} \quad (5)$$

At the last step of the above formulas, matrix notation is taken for simplicity. Here,  $p^\alpha \equiv p$ ,  $g_{\alpha\beta} = \text{diag}(1, -1, -1, -1) \equiv G$ ,  $\Delta^{\alpha\beta} \equiv \Delta$ . When the Lorentz violation matrix  $\Delta^{\alpha\beta}$  is diagonal, e.g.  $\Delta^{\alpha\beta} = \text{diag}(\eta, \xi, \xi, \xi)$ , Eq. (3) becomes  $E^2 = (|\vec{p}|^2(1 - 2\xi + \xi^2) + m^2)/(1 + 2\eta + \eta^2)$ . We write it as the conventional form  $E^2 = |\vec{p}|^2 c_A^2 + (m')^2 c_A^4$ , where  $c_A^2 \equiv (1 - 2\xi + \xi^2)/(1 + 2\eta + \eta^2)$  and  $c_A$  is denoted as the maximal attainable velocity of type  $A$  particles in Refs. [7, 10]. So the effects of the maximal attainable velocity of particles can also be provided by a diagonal Lorentz violation matrix  $\Delta^{\alpha\beta}$  of particles here. As Cohen and Glashow did, we neglect the Lorentz violation of electrons here. Now, Eq. (1) reads

$$\sqrt{m_{\nu_\mu}^2 + \lambda(\Delta_{\nu_\mu, i}, p_{\nu_\mu, i})} \geq \sqrt{m_{\nu_\mu}^2 + \lambda(\Delta_{\nu_\mu, f}, p_{\nu_\mu, f})} + 2m_e. \quad (6)$$

The difference between 4-momenta  $p_{\nu_\mu, i}$  and  $p_{\nu_\mu, f}$  can be notated as a Lorentz transformation  $R$ :  $p_{\nu_\mu, f} = R p_{\nu_\mu, i}$ . When the Lorentz violation matrix  $\Delta_{\nu_\mu, i}$  of the initial muon neutrino and  $\Delta_{\nu_\mu, f}$  of the final/outgoing muon neutrino are same and both are diagonal, e.g.  $\Delta_{\nu_\mu} = \text{diag}(\eta, \xi, \xi, \xi)$  and it is SO(3) invariant, a threshold energy for muon neutrinos can be gotten. Neglecting the mass  $m_{\nu_\mu}$  of muon neutrinos and taking the diagonal form for both  $\Delta_{\nu_\mu, i}$  and  $\Delta_{\nu_\mu, f}$  in Eq. (6), we can get the same threshold energy for initial muon neutrinos as that in Ref. [7]:  $E_{\text{th}} = 2m_e/\sqrt{\delta}$ , where  $\delta \equiv -2(\eta + \xi)$ . If the energy of muon neutrinos is higher than the threshold energy, the process becomes allowed kinematically and the process happens. Now, it is clear that the process is kinematically permitted and a threshold energy of Ref. [7] for initial muon neutrinos can be gotten are based on conditions that  $\Delta_{\nu_\mu, i} = \Delta_{\nu_\mu, f}$  and that  $\Delta_{\nu_\mu, i}, \Delta_{\nu_\mu, f}$  are diagonal, i.e. the Lorentz violation of muon neutrinos is assumed to have a fixed form.

What will happen if the Lorentz violation matrix  $\Delta_{\nu_\mu}$  of muon neutrinos is covariant with the corresponding momentum  $p_{\nu_\mu}$ :  $\Delta_{\nu_\mu, f} = R \Delta_{\nu_\mu, i} R^t$ ? We find that

$$\lambda(\Delta_{\nu_\mu, f}, p_{\nu_\mu, f}) = \lambda(R \Delta_{\nu_\mu, i} R^t, R p_{\nu_\mu, i})$$

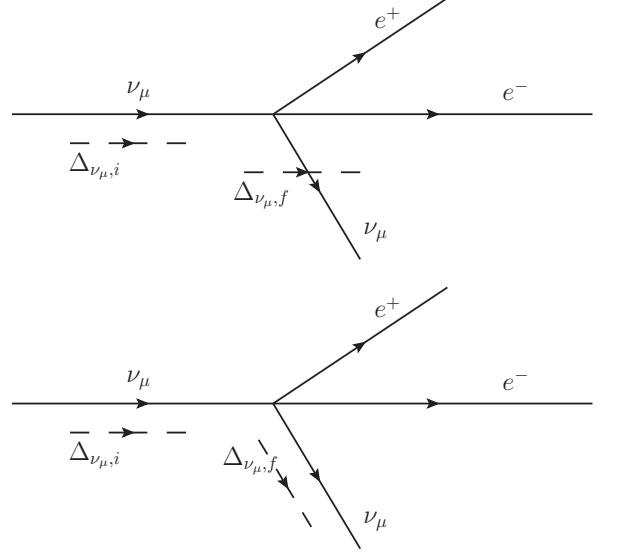


FIG. 1: The fixed form of Lorentz violation for muon neutrinos is illustrated on the upper-panel. The covariant Lorentz violation for muon neutrinos is illustrated on the lower-panel.

$$\begin{aligned} &= -p_{\nu_\mu, i}^t R^t G R \Delta_{\nu_\mu, i}^t R^t G R \Delta_{\nu_\mu, i} R^t G R p_{\nu_\mu, i} \\ &\quad - 2p_{\nu_\mu, i}^t R^t G R \Delta_{\nu_\mu, i} R^t G R p_{\nu_\mu, i} \\ &= -p_{\nu_\mu, i}^t G \Delta_{\nu_\mu, i}^t G \Delta_{\nu_\mu, i} G p_{\nu_\mu, i} - 2p_{\nu_\mu, i}^t G \Delta_{\nu_\mu, i} G p_{\nu_\mu, i} \\ &= \lambda(\Delta_{\nu_\mu, i}, p_{\nu_\mu, i}), \end{aligned}$$

where  $R^t G R = G$  is used. Now we get  $\lambda(\Delta_{\nu_\mu, f}, p_{\nu_\mu, f}) = \lambda(\Delta_{\nu_\mu, i}, p_{\nu_\mu, i})$ . Eq. (6) becomes  $0 \geq 2m_e$ , and it can not be satisfied therefore. Since Eq. (6) is equivalent to  $0 \geq 2m_e$ , Eq. (6) is equivalent to Eq. (2). So the process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  is still forbidden kinematically. With the same discussion, the processes  $\nu_\mu \rightarrow \nu_\mu + \gamma$  and  $\nu_\mu \rightarrow \nu_\mu + \nu_e + \bar{\nu}_e$  are kinematically forbidden too. The corresponding physical pictures of the fixed Lorentz violation and the covariant Lorentz violation are shown in Fig. 1. Since Eq. (3) is Lorentz covariant, it is more natural that the Lorentz violation matrix  $\Delta_{\nu_\mu}$  of muon neutrinos is covariant. On the other hand, for the Cherenkov case, the electromagnetic radiations result from the polarizing of the corresponding medium by the superluminal charged particles, and the response of the medium to the superluminal particles is covariant with the momentum of these particles. It is more appropriate that the Lorentz violation of muon neutrinos is emergent and covariant with muon neutrinos for the process  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  in space-time.

Now we consider the production process of muon neutrinos:  $X \rightarrow \mu + \nu_\mu$ ,  $X = \pi, K$ . The corresponding kinematic limit condition is

$$|p_X| \geq |p_\mu| + |p_{\nu_\mu}|. \quad (7)$$

Without Lorentz violation of muon neutrinos, Eq. (7) is equivalent to  $m_X \geq m_\mu + m_{\nu_\mu}$ , which can be satisfied.

So the process happens in the case of no Lorentz violation. When the Lorentz violation of muon neutrinos is considered, from Eq. (4), Eq. (7) reads

$$m_X \geq m_\mu + \sqrt{m_{\nu_\mu}^2 + \lambda(\Delta_{\nu_\mu}, p_{\nu_\mu})}, \quad (8)$$

where just the Lorentz violation of neutrinos is considered. Even if the Lorentz violation of muon neutrinos in the OPERA experiment is of  $10^{-5}$ , Eq. (8) can still be satisfied, i.e. the production process is still kinematically allowed. The arguments of Ref. [8] that the Lorentz violation of the OPERA forbids the production process mainly results from the fact that a scalar Lorentz violation parameter there can not provide more details about Lorentz violation in space-time. The Lorentz violation is related to all dimensions of space-time. The Lorentz violation matrix  $\Delta_{\nu_\mu}$  of muon neutrinos has tens of degrees of freedom. Calculating  $\lambda(\Delta_{\nu_\mu}, p_{\nu_\mu})$ , we notate that  $p_{\nu_\mu}^\alpha = (E, |\vec{p}|\Omega^1, |\vec{p}|\Omega^2, |\vec{p}|\Omega^3)$ , where  $\Omega^i = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ . We can rewrite  $\lambda(\Delta_{\nu_\mu}, p_{\nu_\mu})$  for simplicity as

$$\lambda(\Delta_{\nu_\mu}, p_{\nu_\mu}) = \alpha E^2 + \beta E|\vec{p}| + \gamma|\vec{p}|^2, \quad (9)$$

where  $\alpha = -2\Delta^{00} + O(\Delta)$ ,  $\beta = 4\Delta^{(0i)}\Omega^i + O(\Delta)$ , and  $\gamma = -2\Delta^{ij}\Omega^i\Omega^j + O(\Delta)$ . So  $\alpha$  is angle independent, and  $\beta$  and  $\gamma$  are angle dependent. The same indices  $i, j$  mean a summation as usual. Around the energy range of OPERA of  $\sim 17$  GeV,  $m_{\nu_\mu} \ll E$ . So the terms of  $m_{\nu_\mu}$  are neglected. From Eq. (4), we get that  $E = E(|\vec{p}|)$ . Then

$$\lambda(\Delta_{\nu_\mu}, p_{\nu_\mu}) = (\alpha + \beta + \gamma)E^2,$$

which is approximated to the first order of  $\alpha, \beta$  and  $\gamma$ . There exist directions  $(\theta_0, \phi_0)$  such that  $\alpha + \beta(\theta_0, \phi_0) + \gamma(\theta_0, \phi_0) \simeq 0$ . So  $\lambda(\Delta_{\nu_\mu}, p_{\nu_\mu})|_{\theta_0, \phi_0} \simeq 0$  MeV<sup>2</sup>, and Eq. (8) is satisfied, i.e. the production processes  $\pi/K \rightarrow \mu + \nu_\mu$  are kinematically allowed in the case of Lorentz violation of  $\sim 10^{-5}$  for the OPERA muon neutrinos. From Eq. (4), we can also get

$$v_{\nu_\mu} \equiv \frac{dE}{d|\vec{p}|} = 1 + \frac{1}{2}(\alpha + \beta + \gamma) + f(\alpha, \beta, \gamma), \quad (10)$$

which is approximated to the second order. And  $f(\alpha, \beta, \gamma) = (3\alpha^2 + \beta^2 - \gamma^2 + 4\alpha\beta + 2\alpha\gamma)/8$ . Muon neutrinos propagate to Gran Sasso from CERN in direction  $(\theta_1, \phi_1)$ . Generally,  $(\theta_1, \phi_1) \neq (\theta_0, \phi_0)$ . Then Eq. (10) becomes

$$\begin{aligned} v_{\nu_\mu}|_{\theta_1, \phi_1} &= 1 + \frac{1}{2}(\delta\beta(\theta_1, \phi_1) + \delta\gamma(\theta_1, \phi_1)) \\ &+ f(\alpha, \beta(\theta_1, \phi_1), \gamma(\theta_1, \phi_1)) \\ &= 1 + 2\Delta^{(0i)}\delta\Omega^i(\theta_1, \phi_1) - \Delta^{ij}\delta(\Omega^i\Omega^j) + f|_{\theta_1, \phi_1}, \end{aligned} \quad (11)$$

where  $\delta\Omega^i(\theta_1, \phi_1) \equiv \Omega^i(\theta_1, \phi_1) - \Omega^i(\theta_0, \phi_0)$ .  $2\delta\Omega^i(\theta_1, \phi_1)$  and  $\delta(\Omega^i\Omega^j)$  are  $O(1)$ , so  $\Delta^{(0i)} \sim 10^{-5}$  and  $\Delta^{ij} \sim 10^{-5}$  to

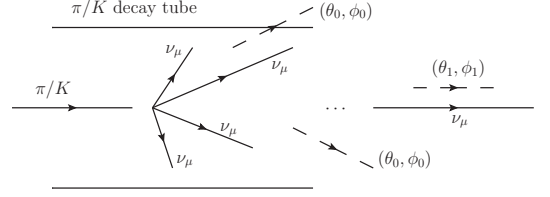


FIG. 2: A brief illustration of the OPERA muon neutrino beam.

give the OPERA speed anomaly of  $\sim 10^{-5}$ . If  $(\theta_1, \phi_1) = (\theta_0, \phi_0)$ , then  $f|_{\theta_1, \phi_1} \sim 10^{-5}$ . The illustration is shown in Fig. 2.

It has been reported [11] by the ICARUS Collaboration that there is no evidence for the analogous Cherenkov radiation of muon neutrinos from CERN to the LNGS, where the OPERA experiment is also performed. If taking the arguments of Cohen-Glashow and Bi *et al.* as true, then one must refute the OPERA result as Ref. [11] did. However, we take the ICARUS result as a support of our argument on the forbidding of these Cherenkov-like processes, rather than a refutation of the OPERA result. Therefore our argument can accommodate both the OPERA and the ICARUS experiments, whereas one must refute the OPERA result or the ICARUS result based on the arguments for these Cherenkov-like processes.

In summary, we get that even if the Lorentz violation parameters  $\Delta_{\nu_\mu}^{0i}$ ,  $\Delta_{\nu_\mu}^{i0}$  and  $\Delta_{\nu_\mu}^{ij}$  are  $\sim 10^{-5}$  for the OPERA experiment, muon neutrinos are still able to be produced through processes  $\pi/K \rightarrow \mu + \nu_\mu$ . After being generated, muon neutrinos are still able to be free from the Cherenkov analogous processes  $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$  and etc, propagating from CERN to Gran Sasso, if the Lorentz violation is covariant with the muon neutrino when the Cherenkov-like processes happen. So the argument of Ref. [7] is just among one of the theoretically assumption-dependent possibilities. The “obvious” forbidding to the OPERA experiment of muon neutrinos is not so obvious. Our argument not only manifest the theoretical rationality for the Lorentz violation of muon neutrinos in the OPERA experiment, but can also accommodate both the OPERA and the ICARUS results. On the other hand, we agree that the superluminality of muon neutrinos in the OPERA experiment still needs to be checked by further experiments.

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